Tractable Computational Methods for Finding Nash Equilibria of Perfect-Information Position Auctions

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Motivation

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Main message: preliminary, but it works

Outline

- Auctions & Model
- Action-Graph Games
- Auctions as AGGs
- Computational Experiments
- Economic Experiments

Types of Position Auctions

- Dimensions:
  - Generalized First Price vs. Generalized Second Price
  - Pay-per-click vs. Pay-per-impression
  - Weighted vs. Unweighted:
    - “Effective Bid”: bid * weight
    - Ads ranked by effective bid
    - Payment: effective bid / weight
- Current Usage (Google,Microsoft,Yahoo!):
  - Weighted, Per-Click, GSP

Model of Auction Setting

Full-information, one-shot game [Varian, 2007; Edelman, Ostrovsky, Schwarz, 2006 ("EOS")]

	Weights	**CTR across positions**	**CTR across bidders**	**Value per Click**	**Bid Amounts**
[EOS]	Always 1	Decreasing	Constant	One value per bidder	Continuous
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Our model	Arbitrary	Arbitrary	Arbitrary	Arbitrary	Discrete

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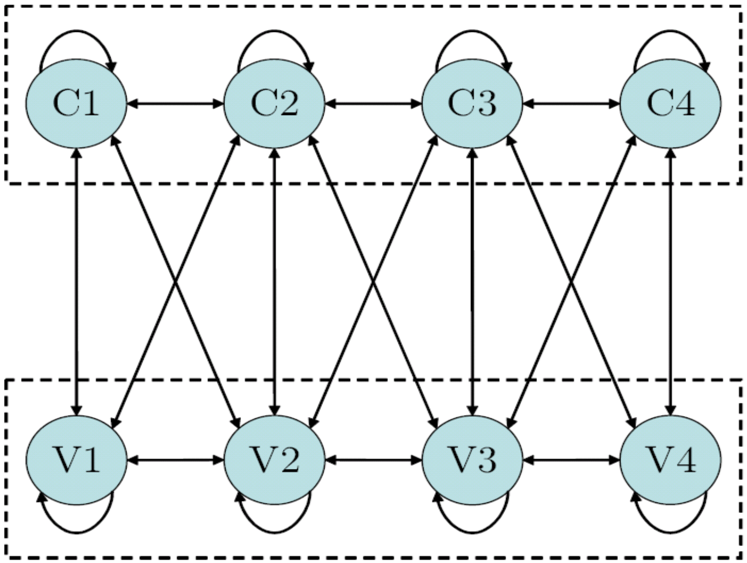
What are AGGs?

- Action Graphs:
  - Each node represents an action.
  - Arcs indicate payoff dependencies.



- [Bhat & Leyton-Brown, 2004; Jiang & Leyton-Brown, 2006]

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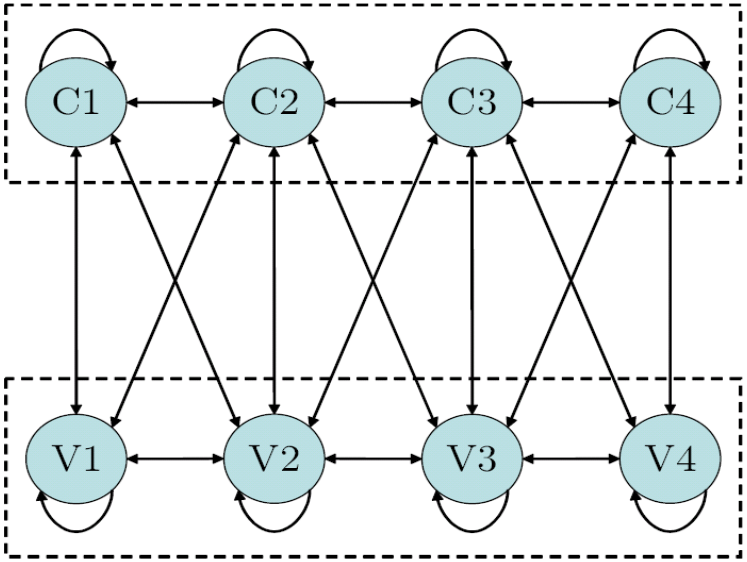
  - Each node represents an action.
  - Arcs indicate payoff dependencies.
  - “Function Nodes” increase sparsity.



- [Bhat & Leyton-Brown, 2004; Jiang & Leyton-Brown, 2006]

Why Use AGGs? [Bhat & Leyton-Brown, 2004]

- Small: Compact representation of a one-shot, full-information game
  - Frequently polynomial in n

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- Small: Compact representation of a one-shot, full-information game
  - Frequently polynomial in n
- Fast: Dynamic programing can compute expected utility in ~$O(an^{i+1})$ [Jiang & Leyton-Brown, 2006]
  - Plug into existing equilibrium solvers (e.g. simplicial subdivision [van der Laan, Talman, and van Der Heyden, 1987] or GNM [Govindan, Wilson, 2003]) for exponential speedup

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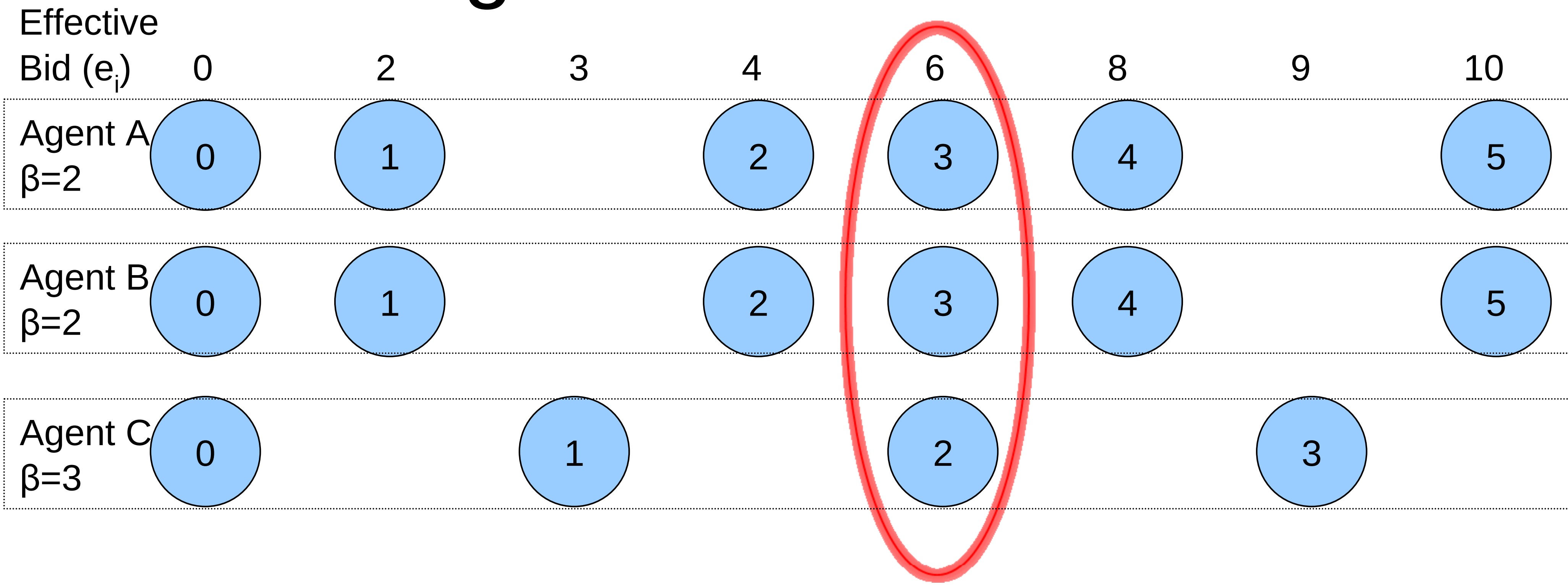
Weighted GFP as AGG

Agent A
$\beta=2$

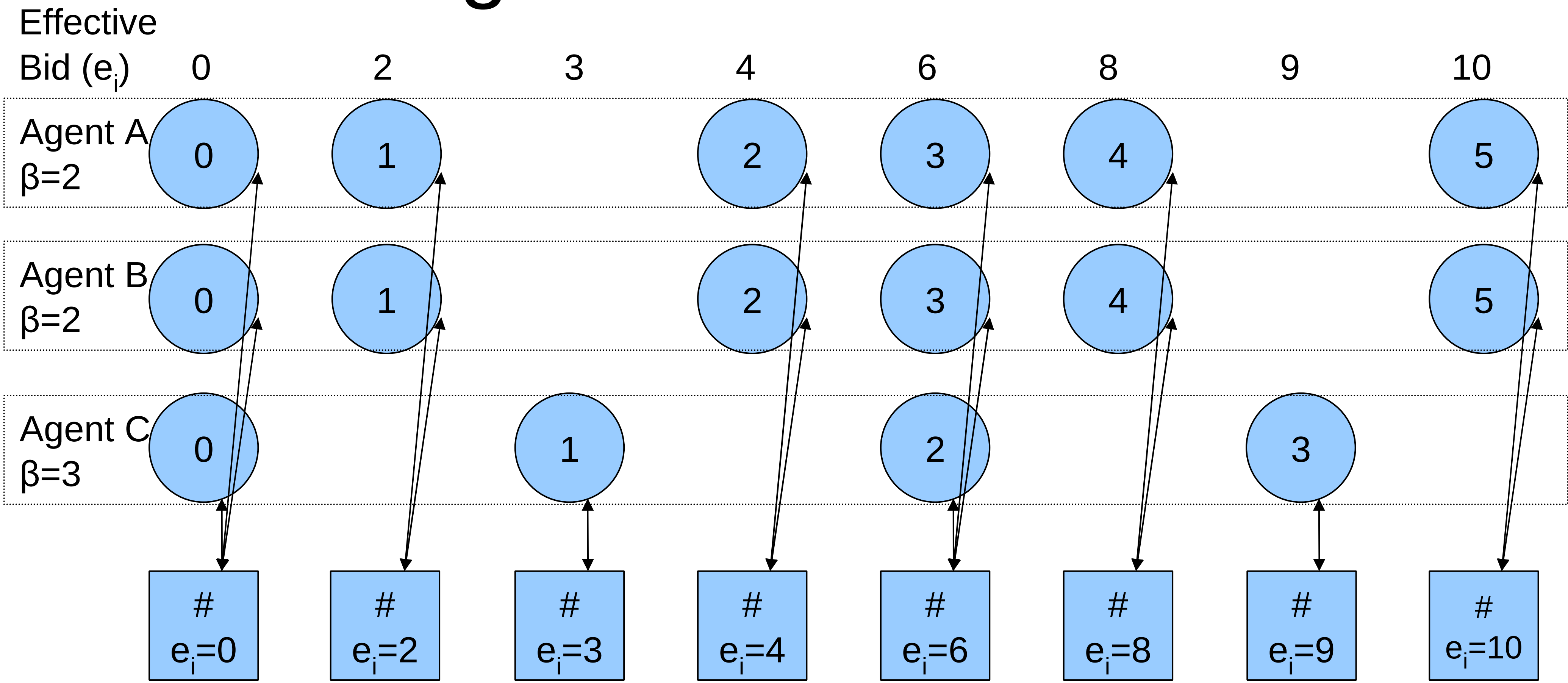
Agent B
$\beta=2$

Agent C
$\beta=3$

Weighted GFP as AGG



Weighted GFP as AGG



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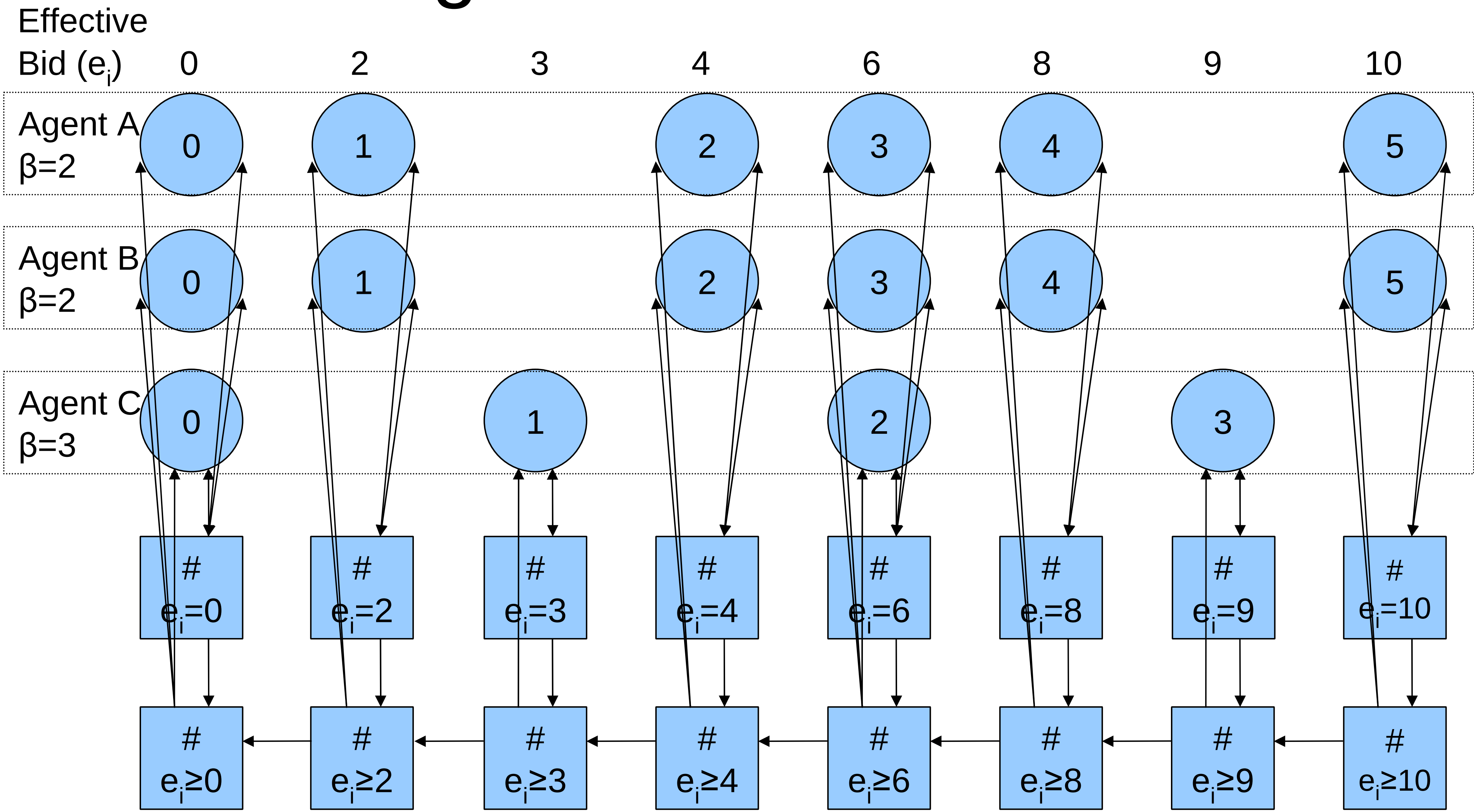

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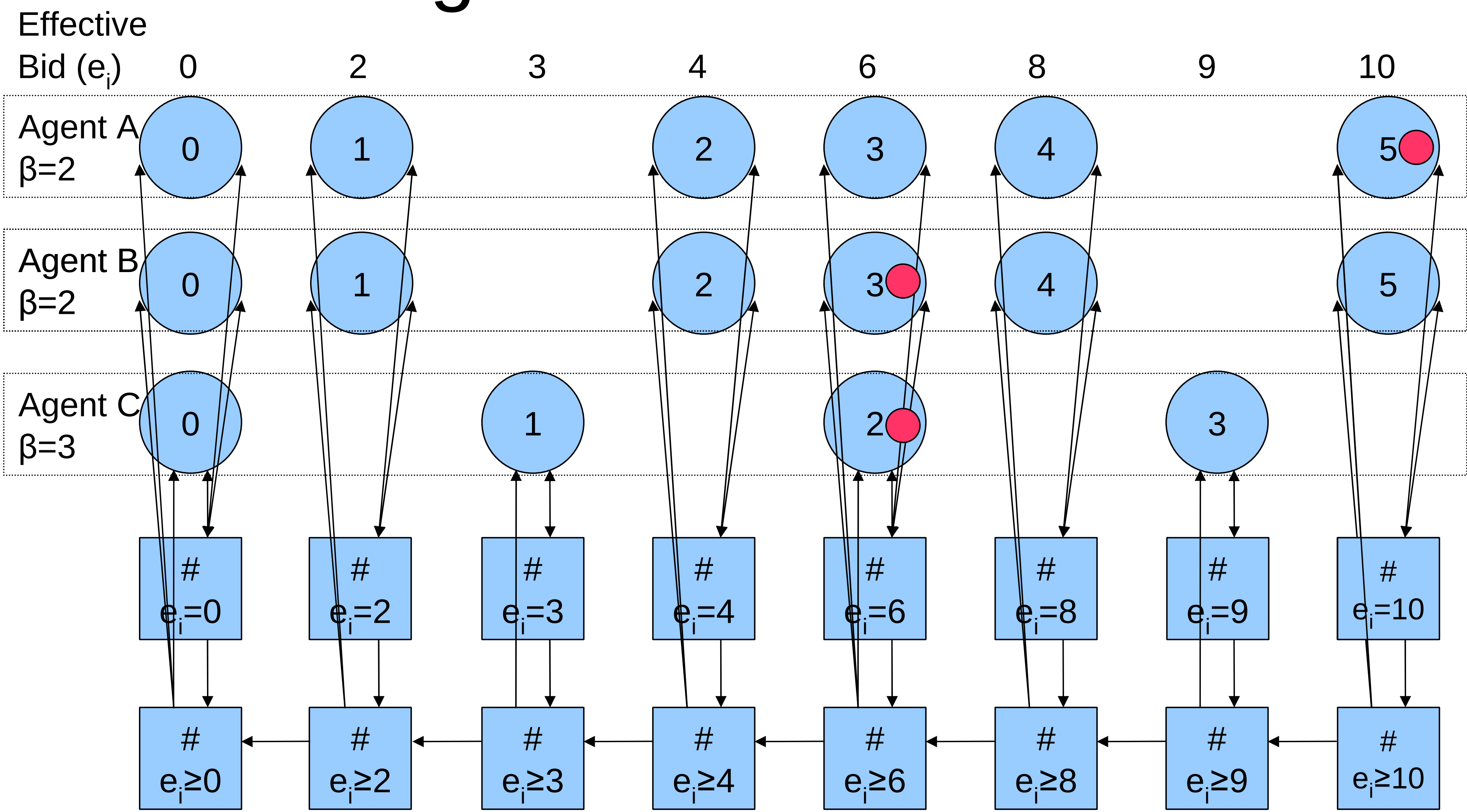

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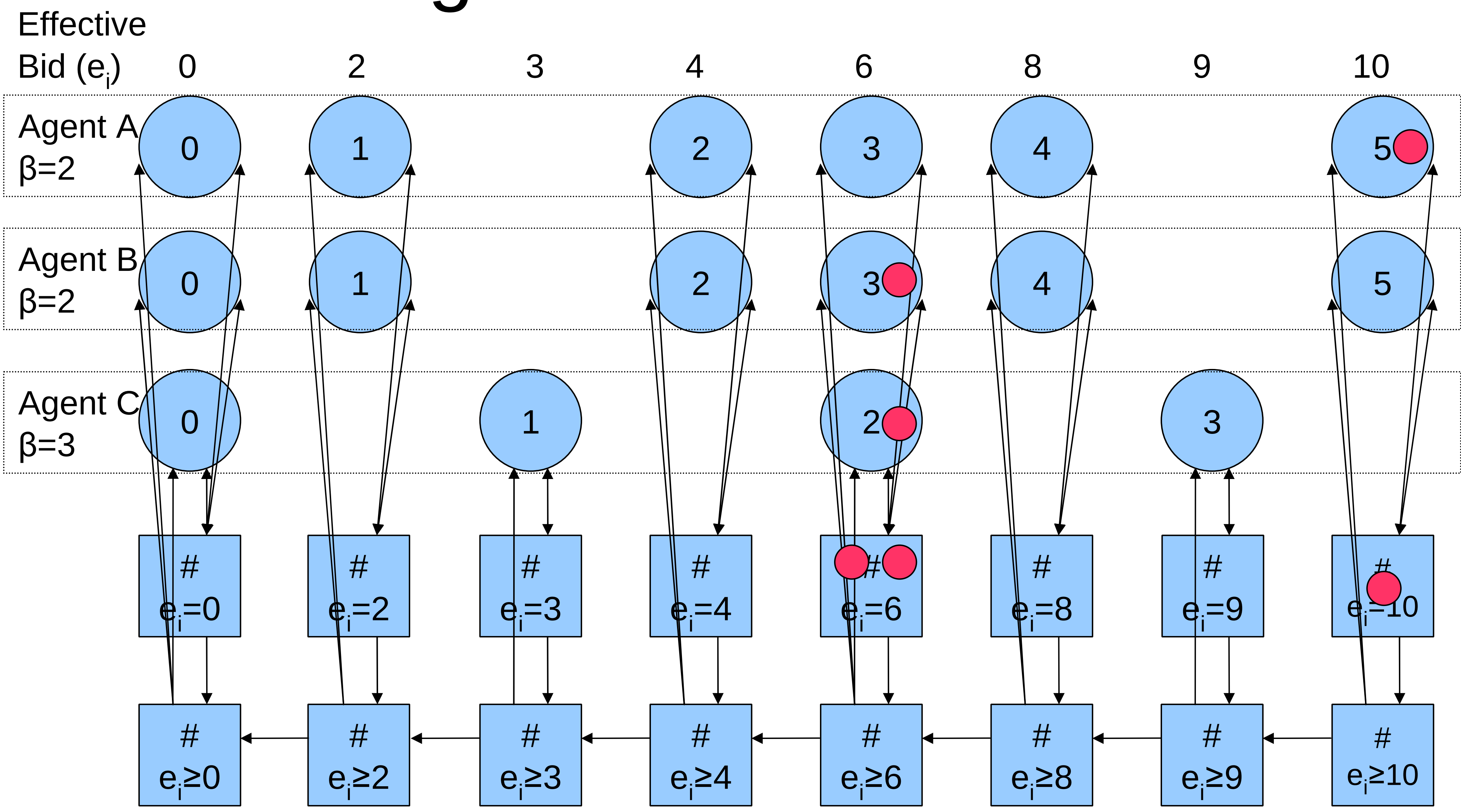

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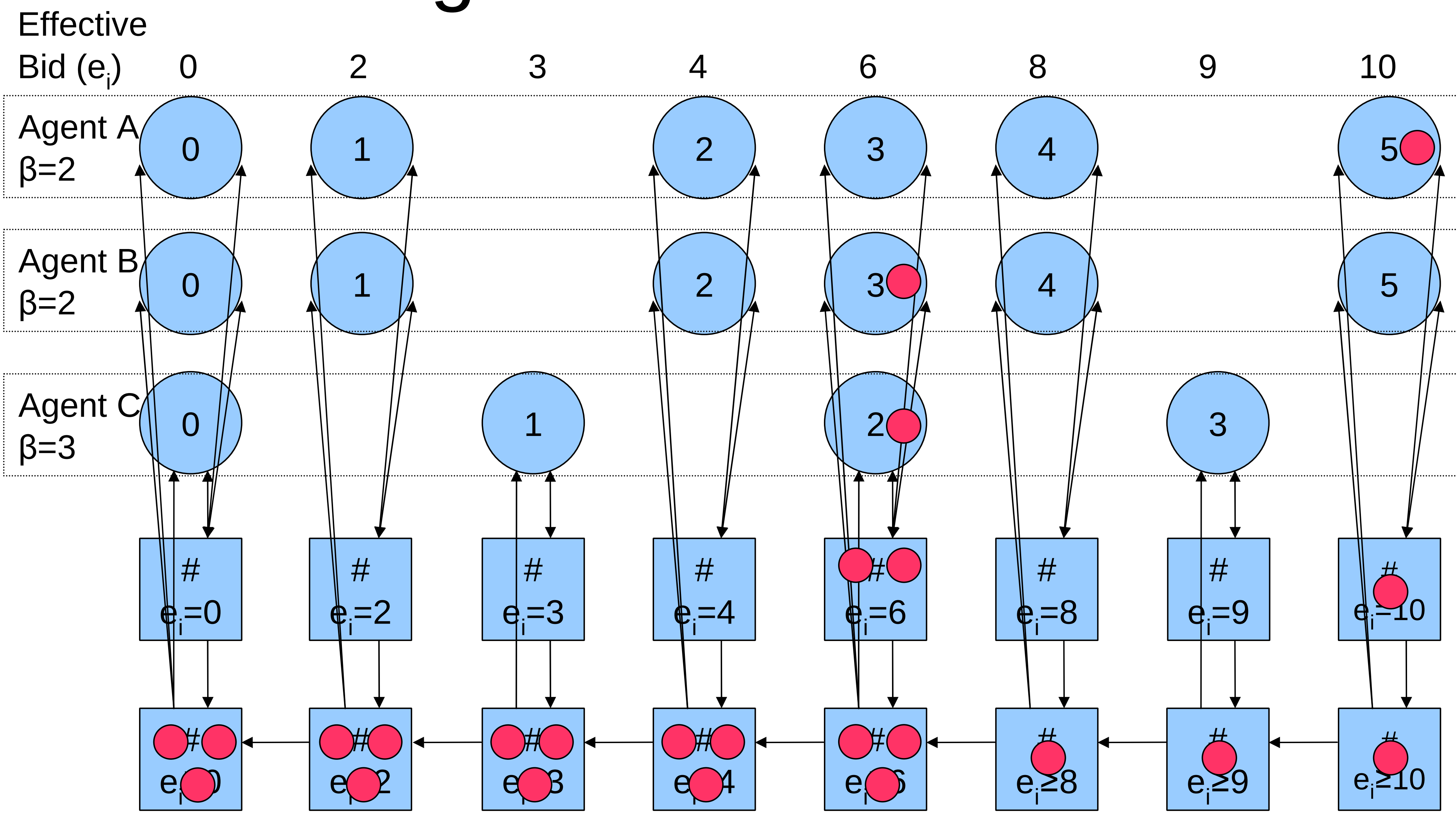
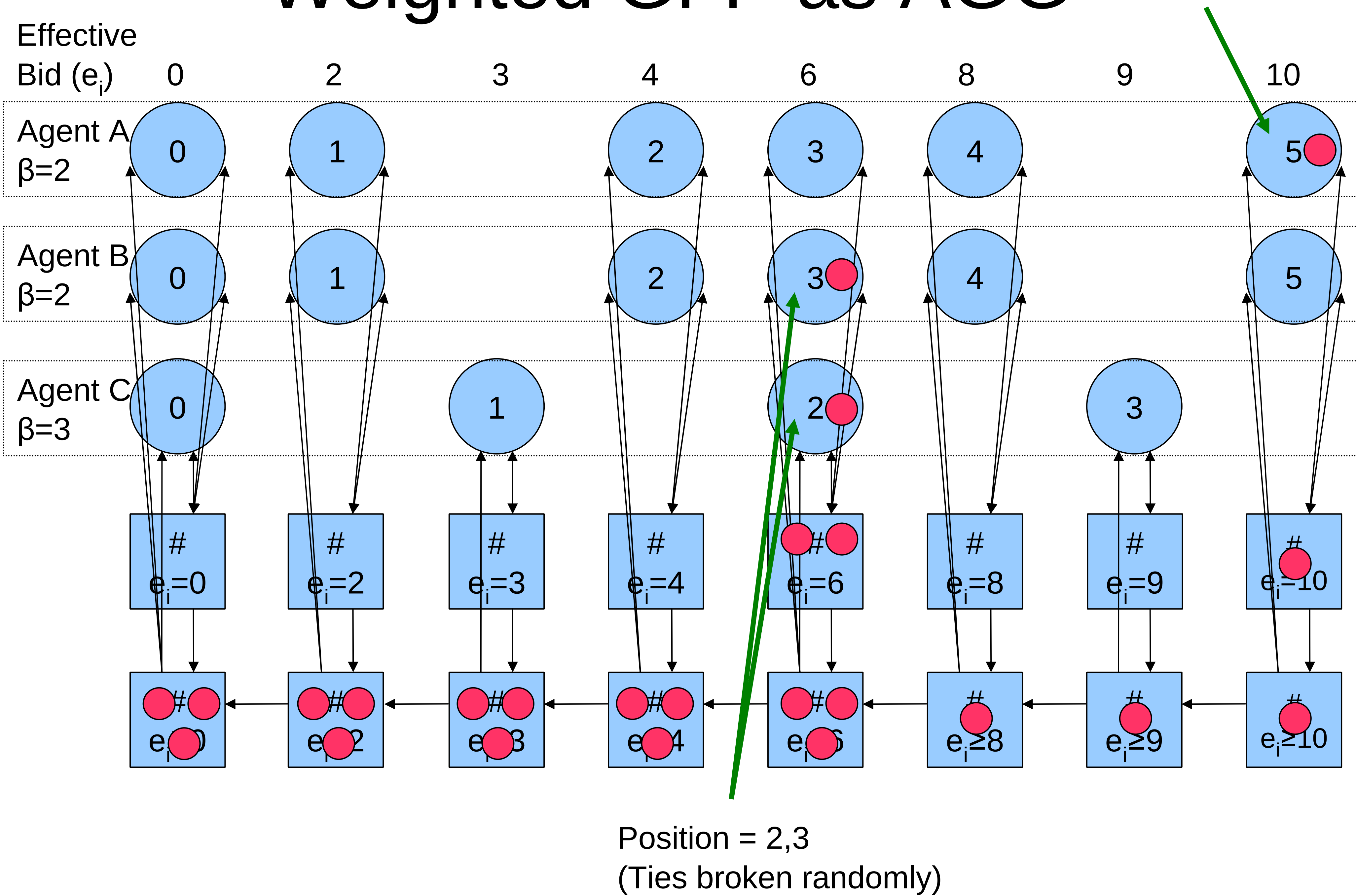


Weighted GFP as AGG

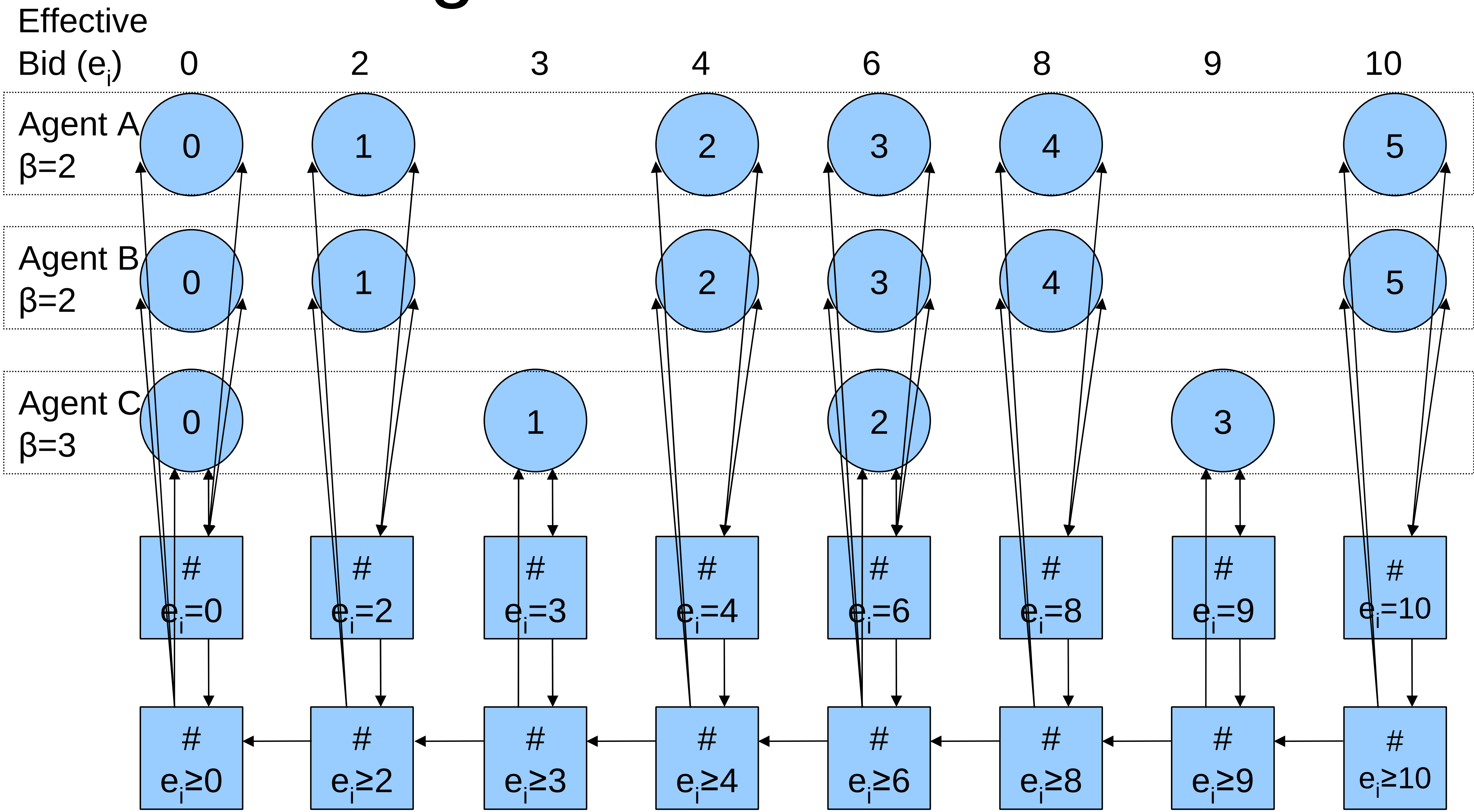
Position = 1



Representing GSP

- Start from a GFP graph
  - same method of computing a bidder's position
- We need to add new nodes to compute prices

Weighted GSP as AGG



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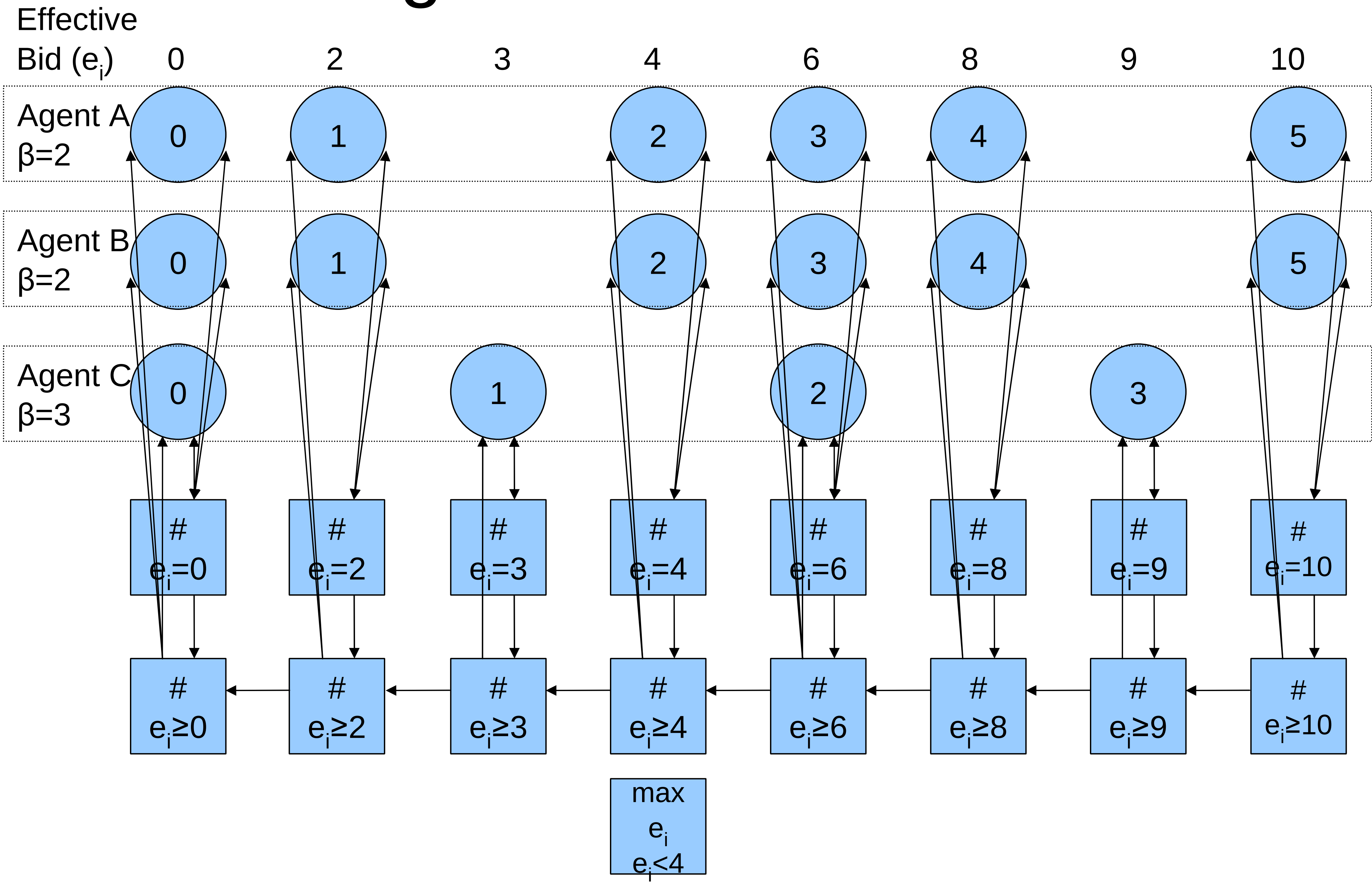

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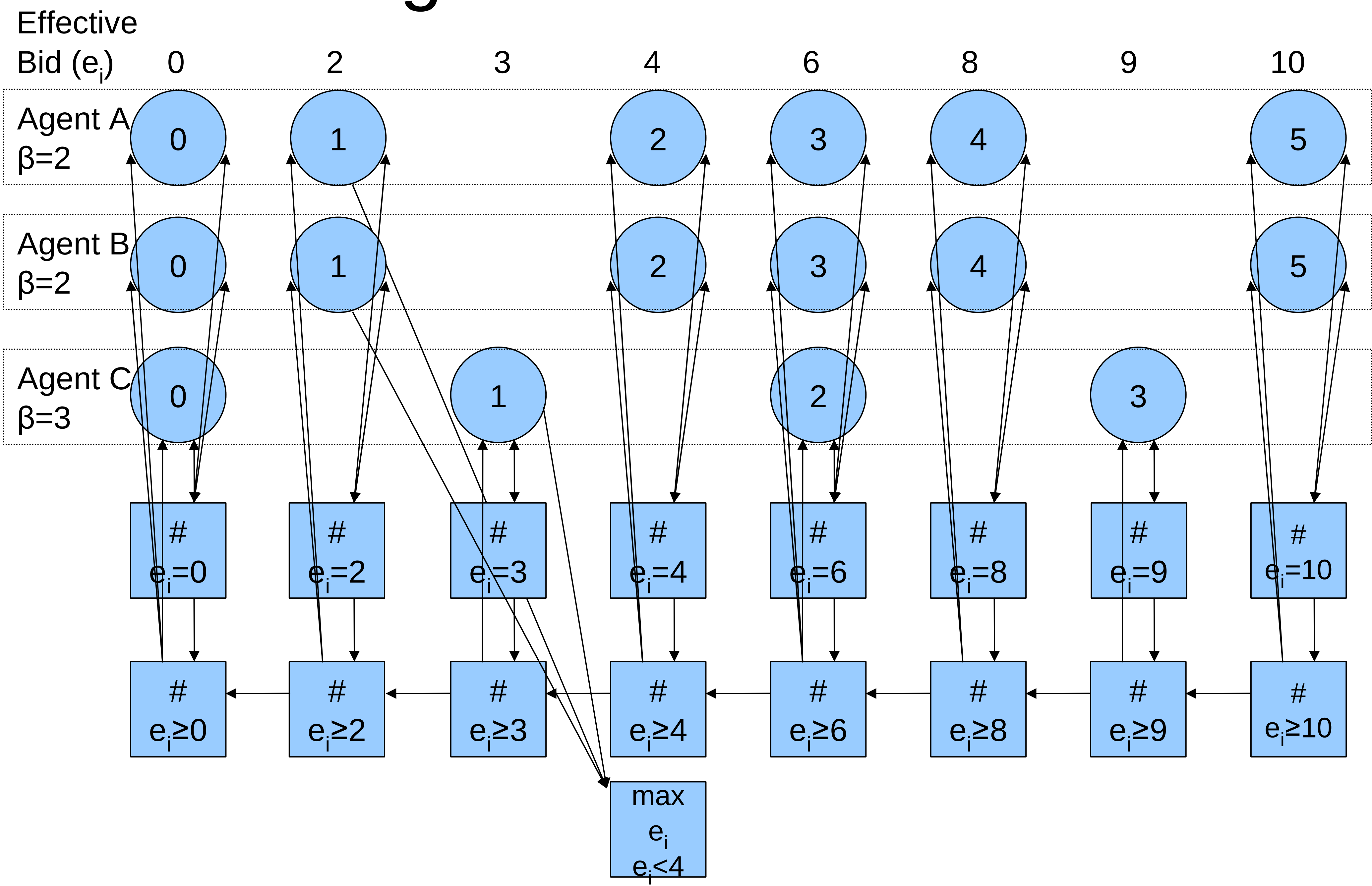

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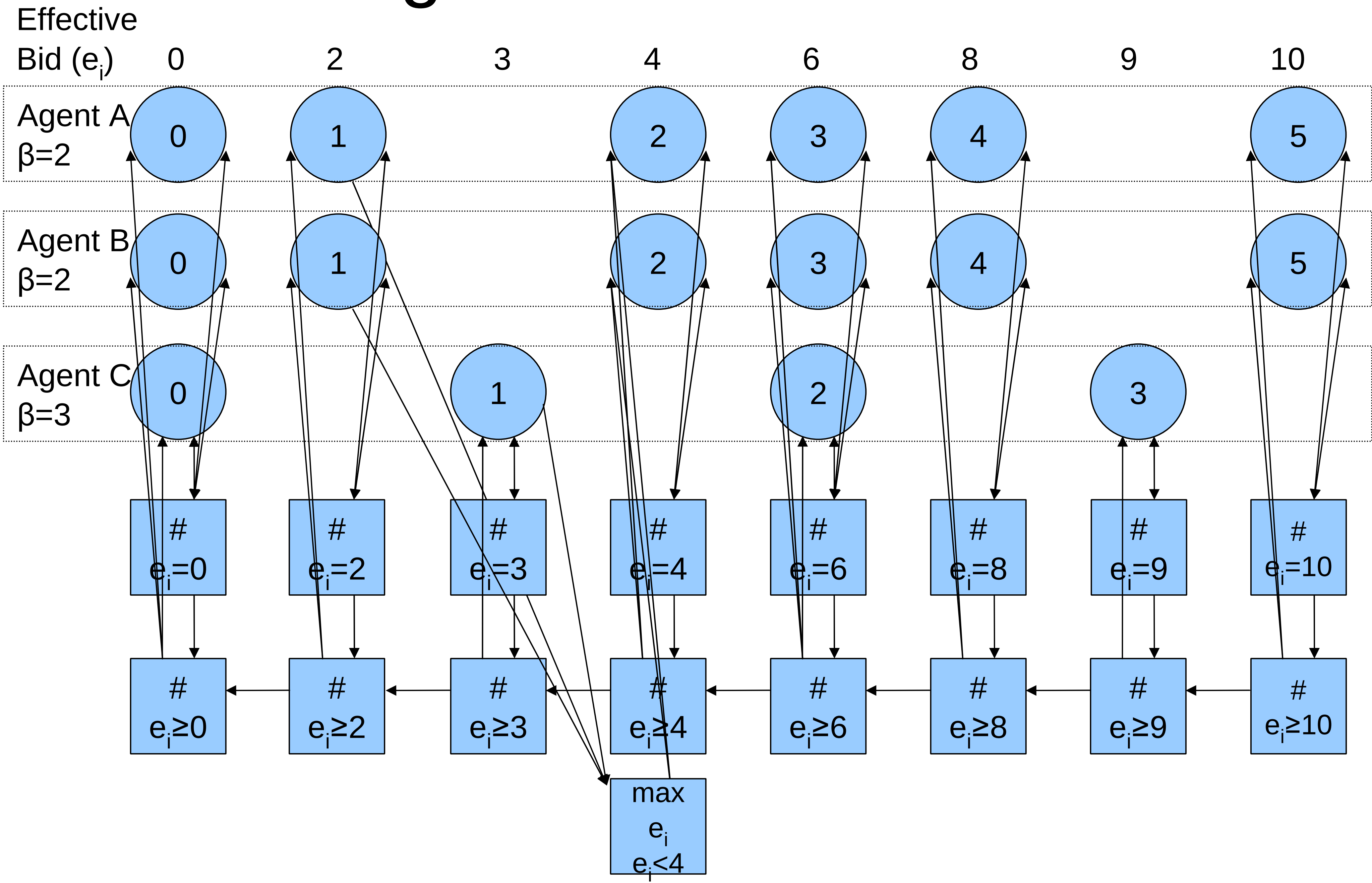

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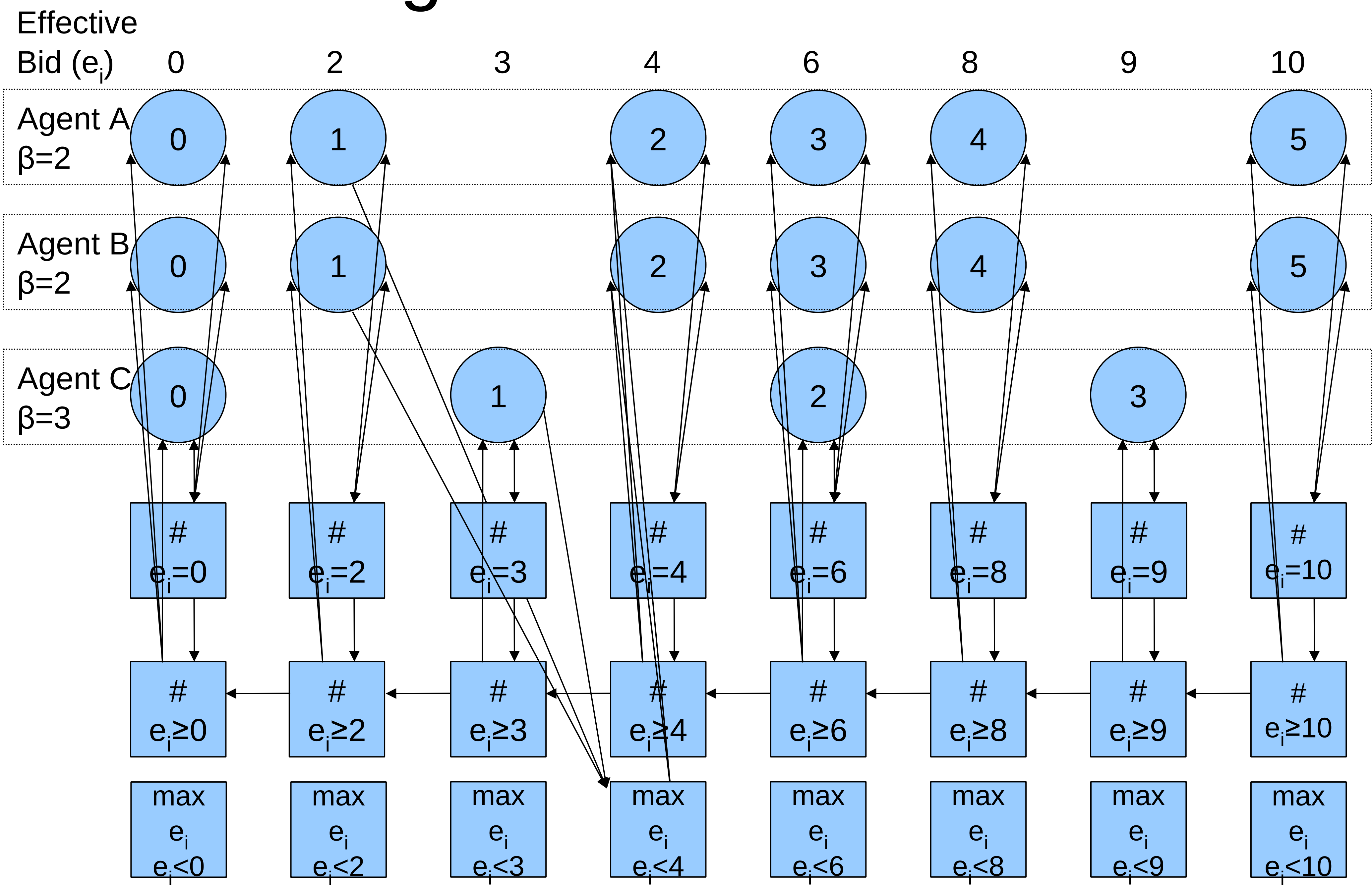

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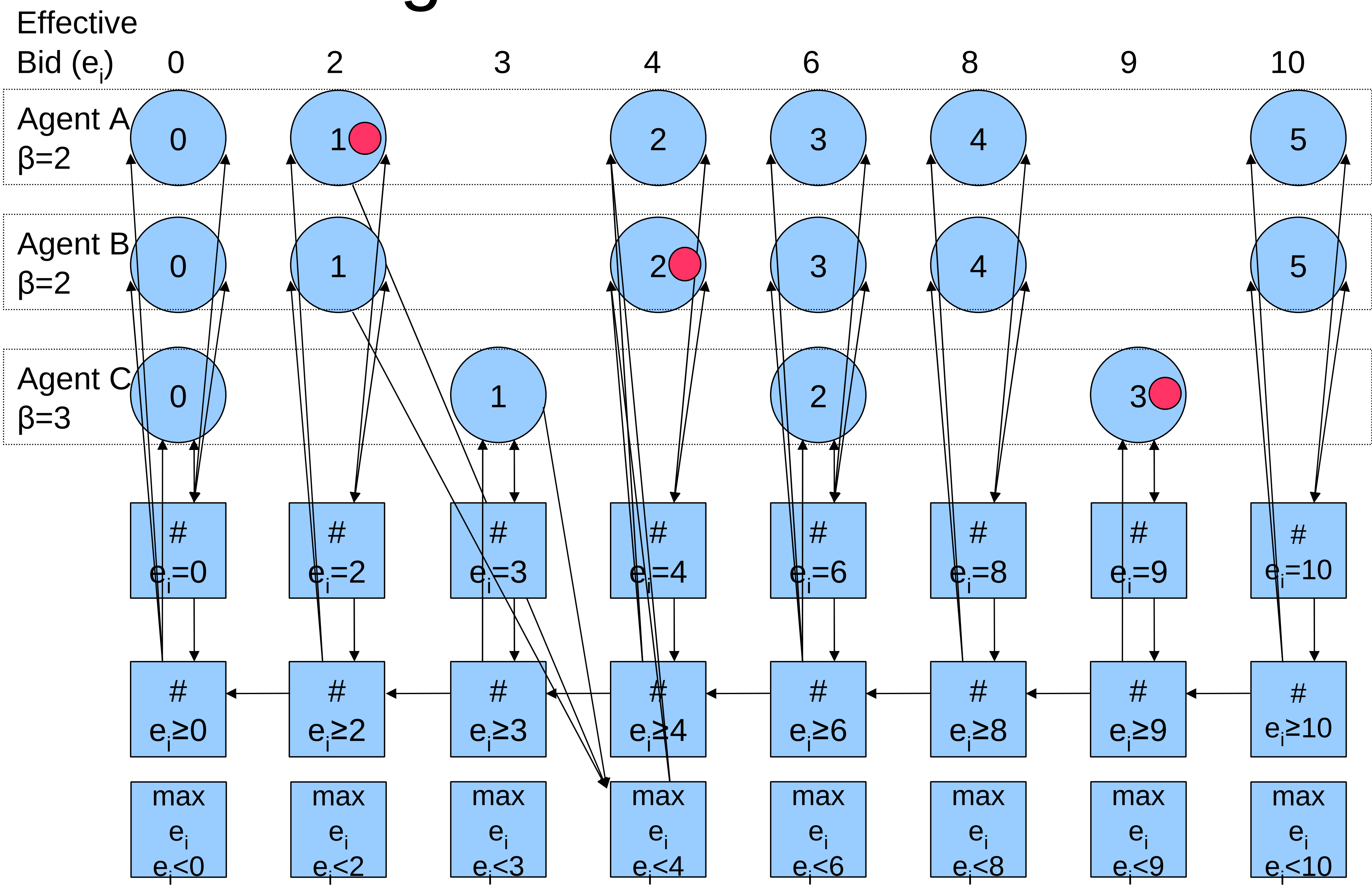

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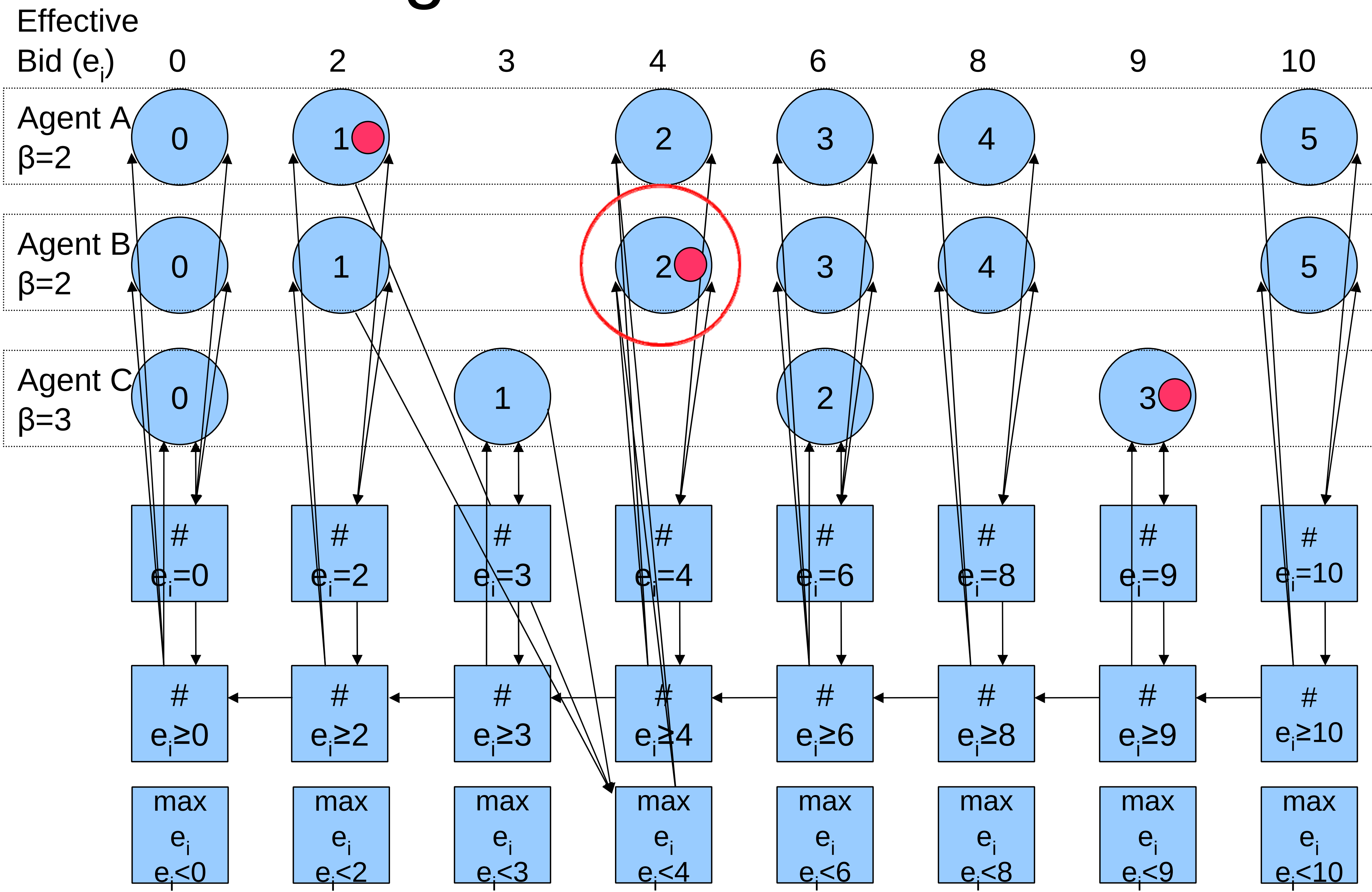

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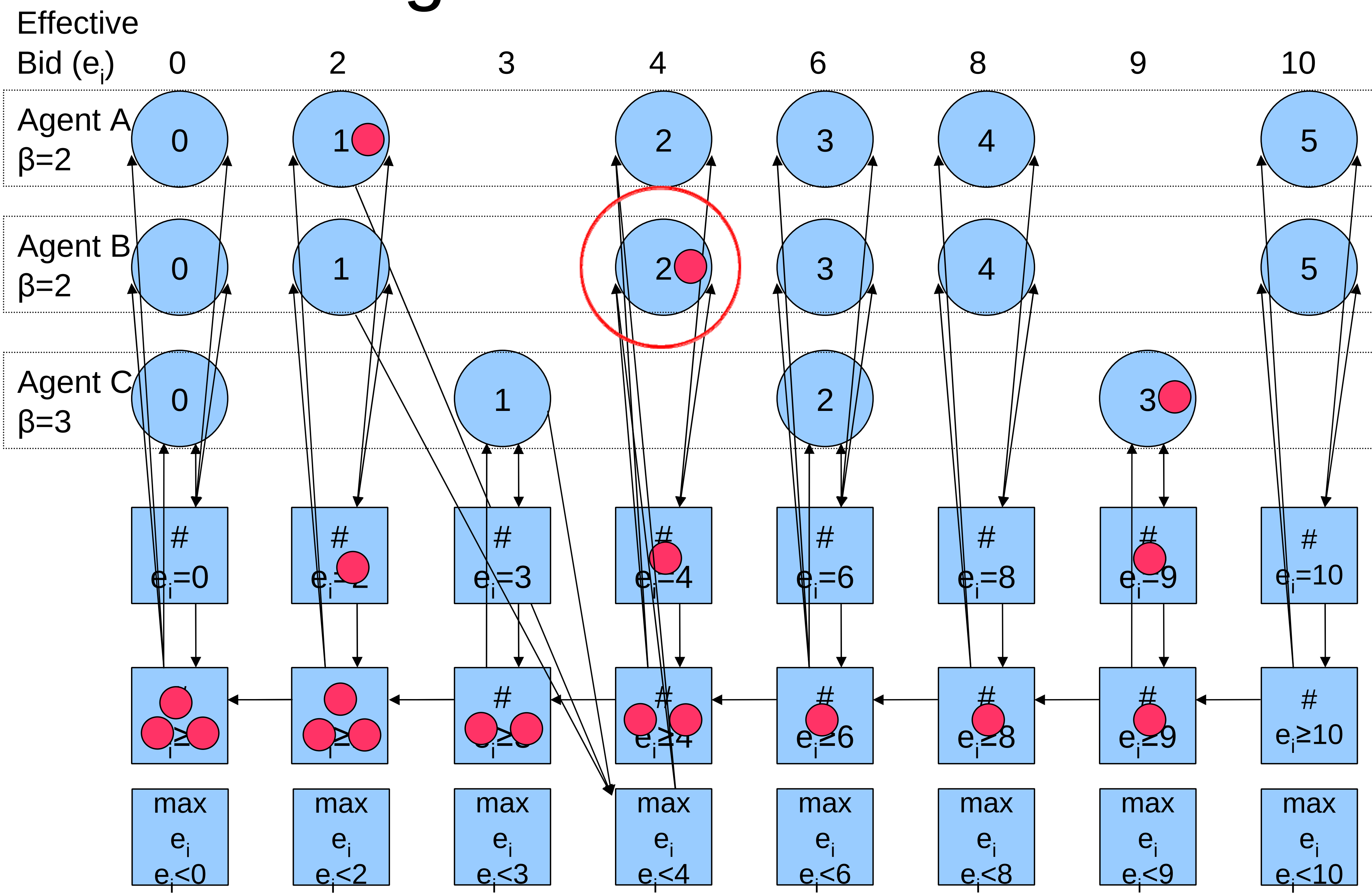

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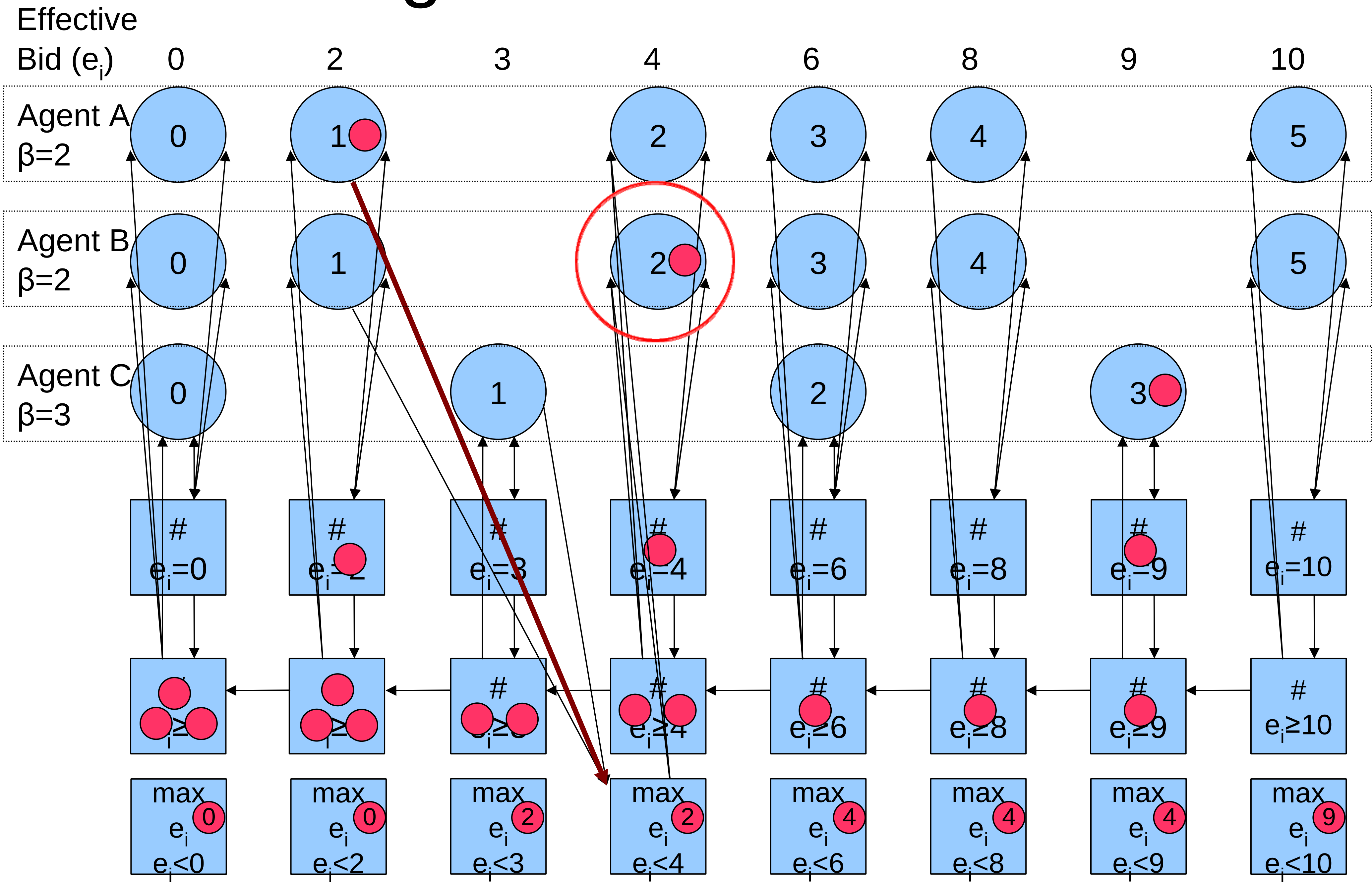
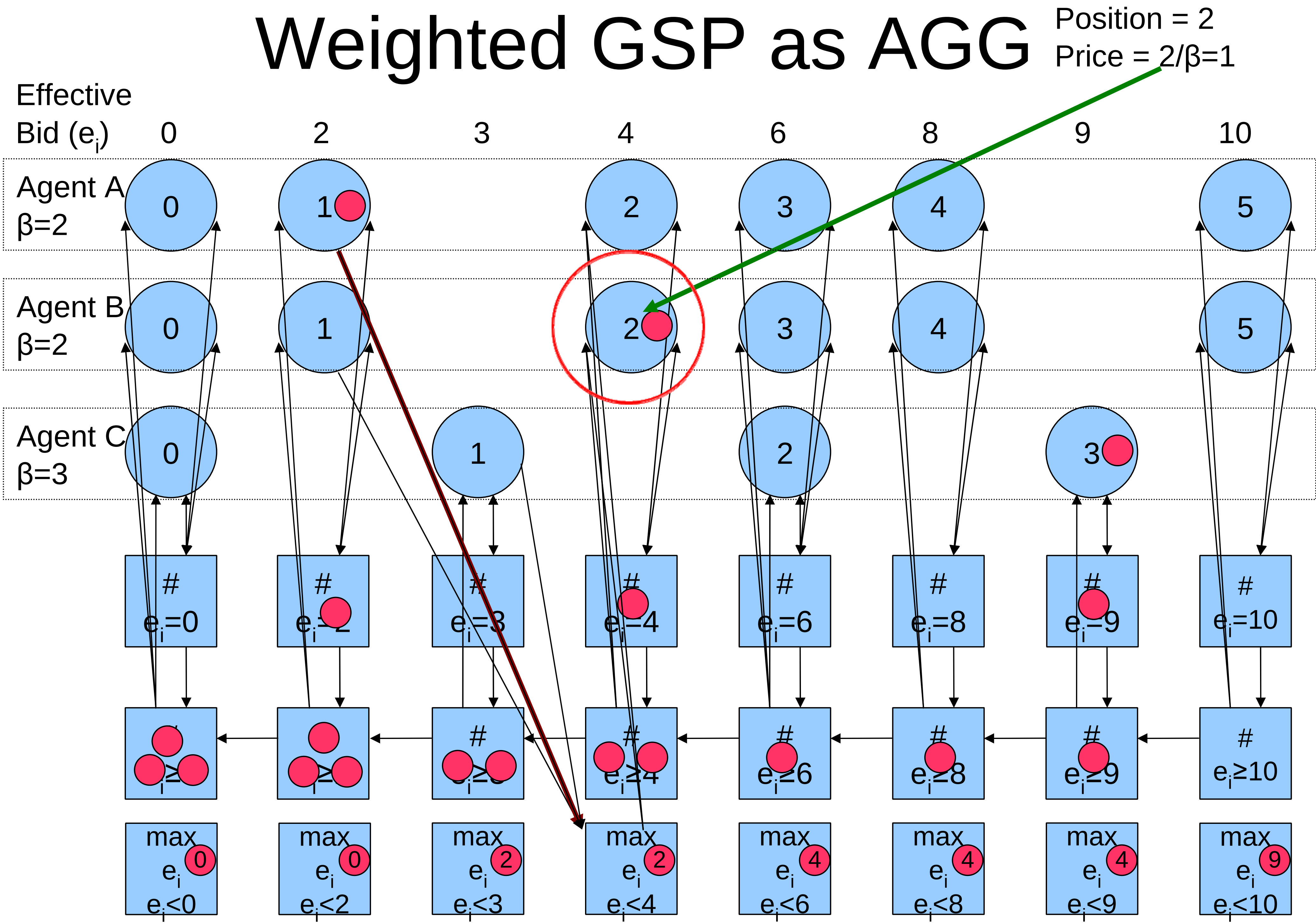


Weighted GSP as AGG



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Why Instantiate [Varian]?

- Validate by comparing with Varian's analytical results for weighted, pay-per-click GSP
  - and obtain computational results on a model of independent interest
- Obtain novel economic results
  - “Apples-to-apples” comparison: how do different auctions perform given identical preferences?
- Most appropriate model is still an open question

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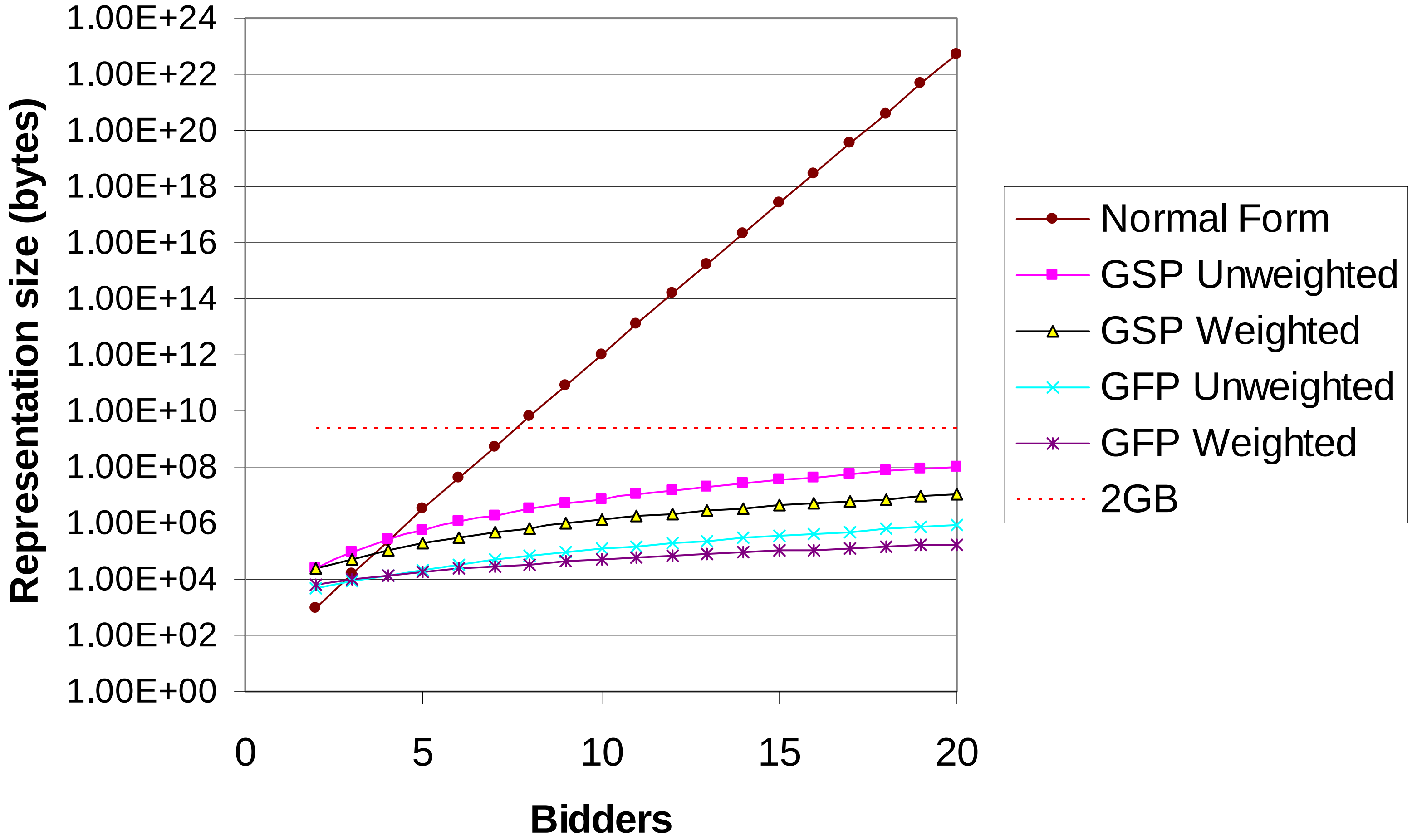
Problem Distribution	Uniform[0,1]	Uniform[0,1] * CTR of higher slot	Proportional to Weight (“Separable”)	One value per bidder: Uniform[0,1]	Discrete

Experimental Setup

- 10 bidders, 5 slots
- Integer bids between 0 and 10
- For pay-per-click, normalize value/click:
  - Scale $\max_i$ $value_i$ to 10, then scale other values proportionately
    - to use full range of discrete bid amounts
- For pay-per-impression, normalize value/impression.

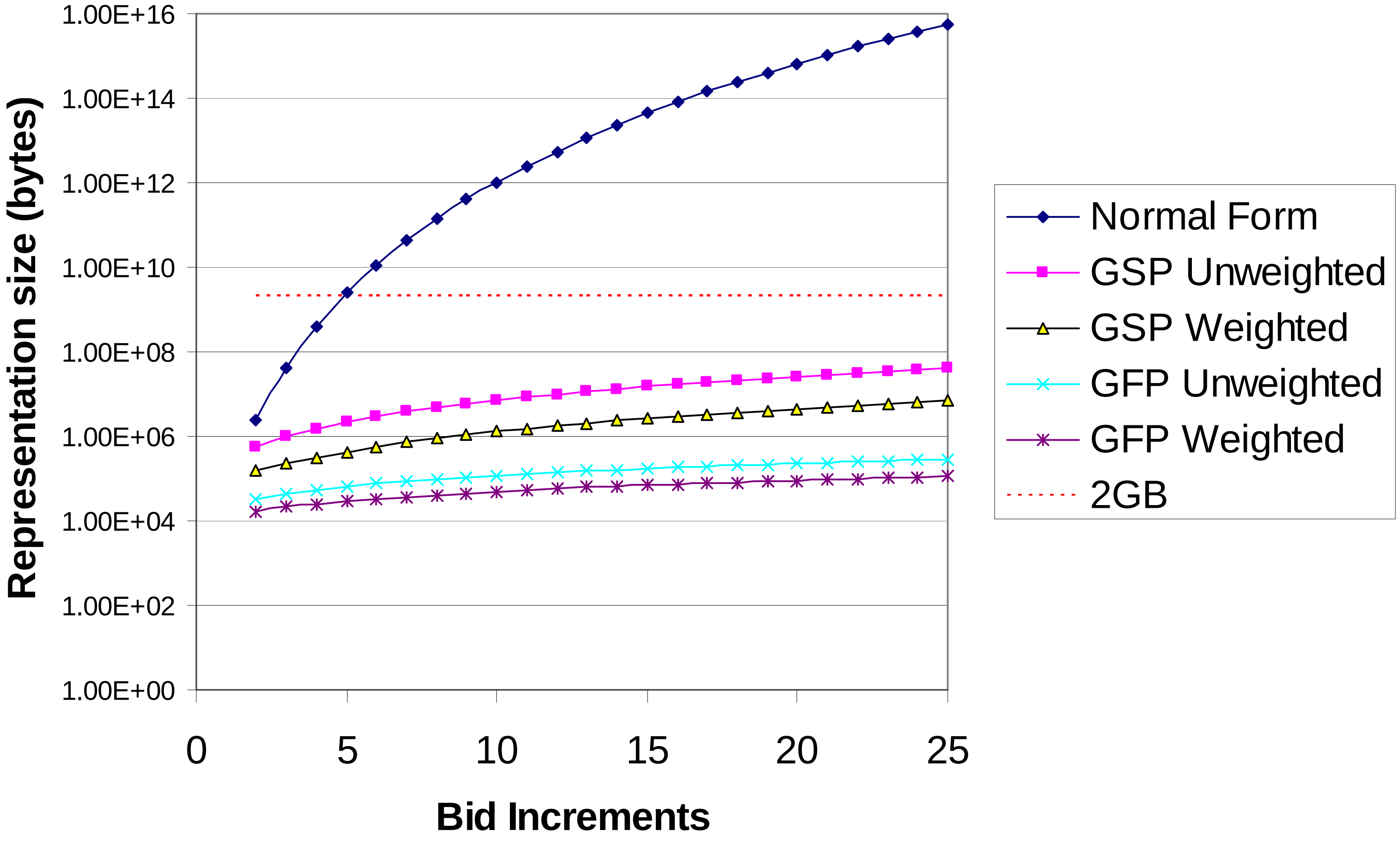
Size Experiments: Players

Integer bids: 0 to 10



Size Experiments: Bid Increments

10 bidders

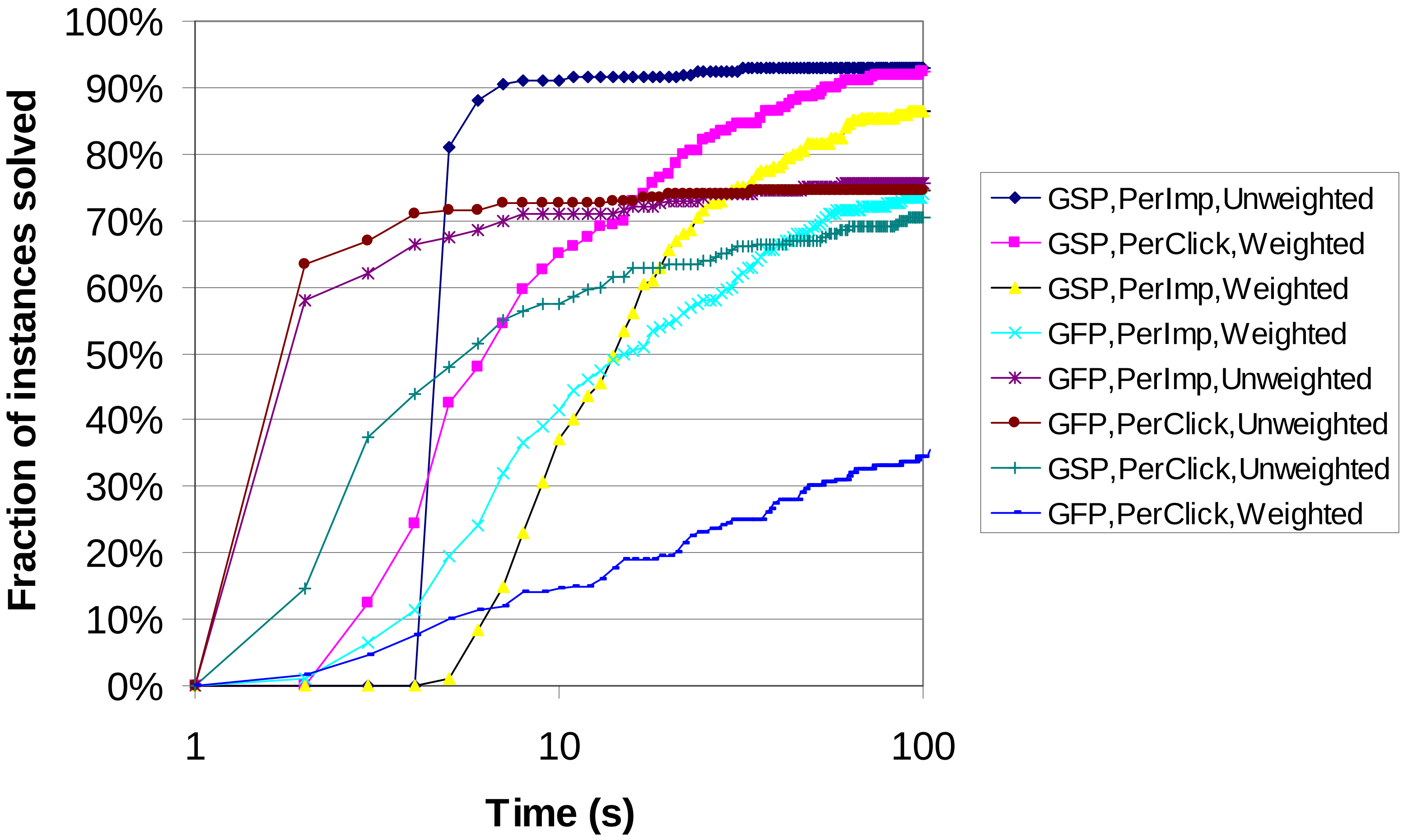


Runtime Experiments: Test-bed

- Environment:
  - Intel Xeon 3.2GHz, 2MB cache, 2GB RAM
  - Suse Linux 10.1
- Solver software:
  - Gambit [McKelvey, McLennan, Turocy, 2007] implementation of simplicial subdivision "simpdiv" [van der Laan, Talman, and van Der Heyden, 1987], AGG-specific dynamic programming inner loop[1] [Jiang & Leyton-Brown, 2006]

1. http://www.cs.ubc.ca/~jiang/agg/

Runtime Experiments: Results



* much longer experiments are ongoing…

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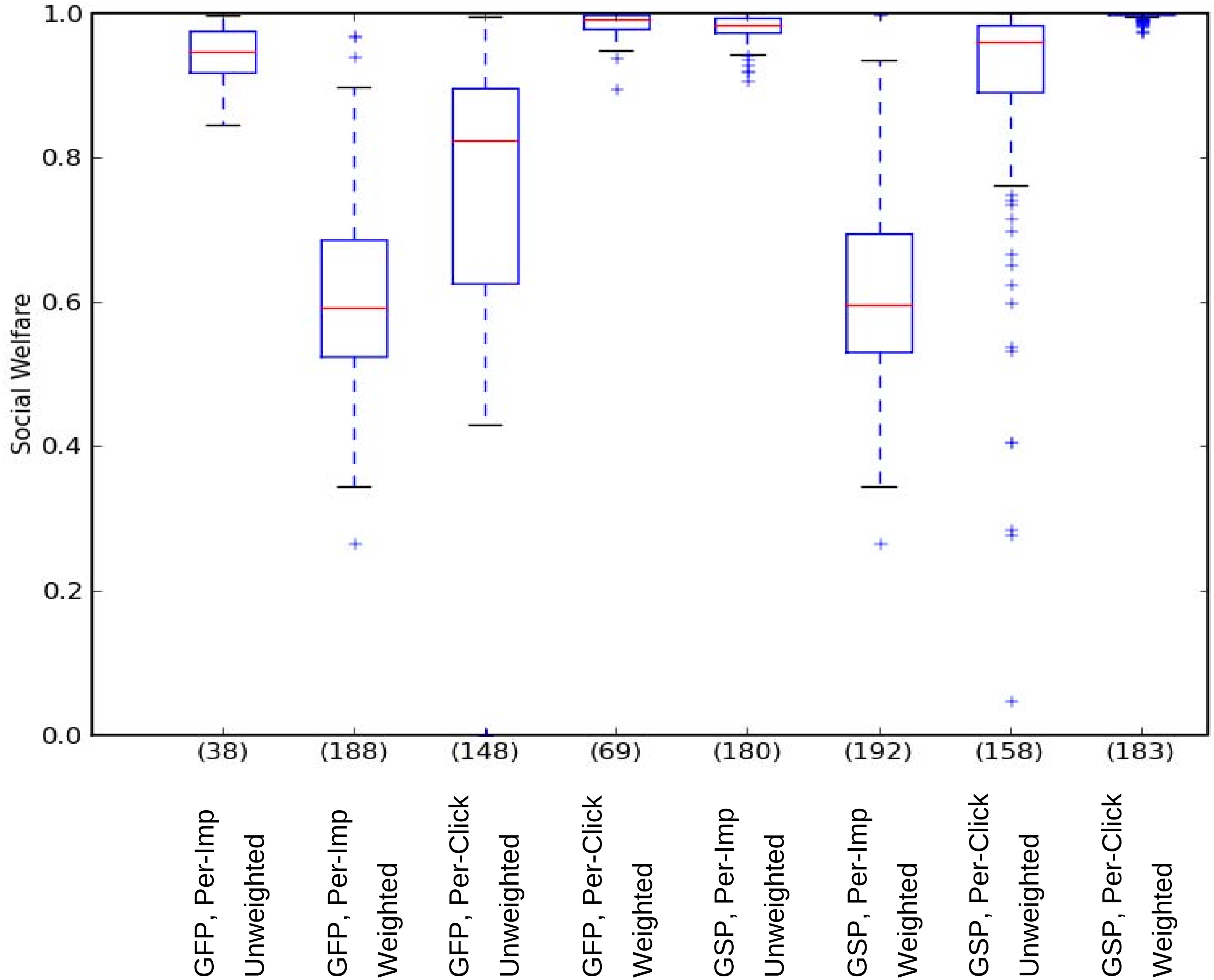
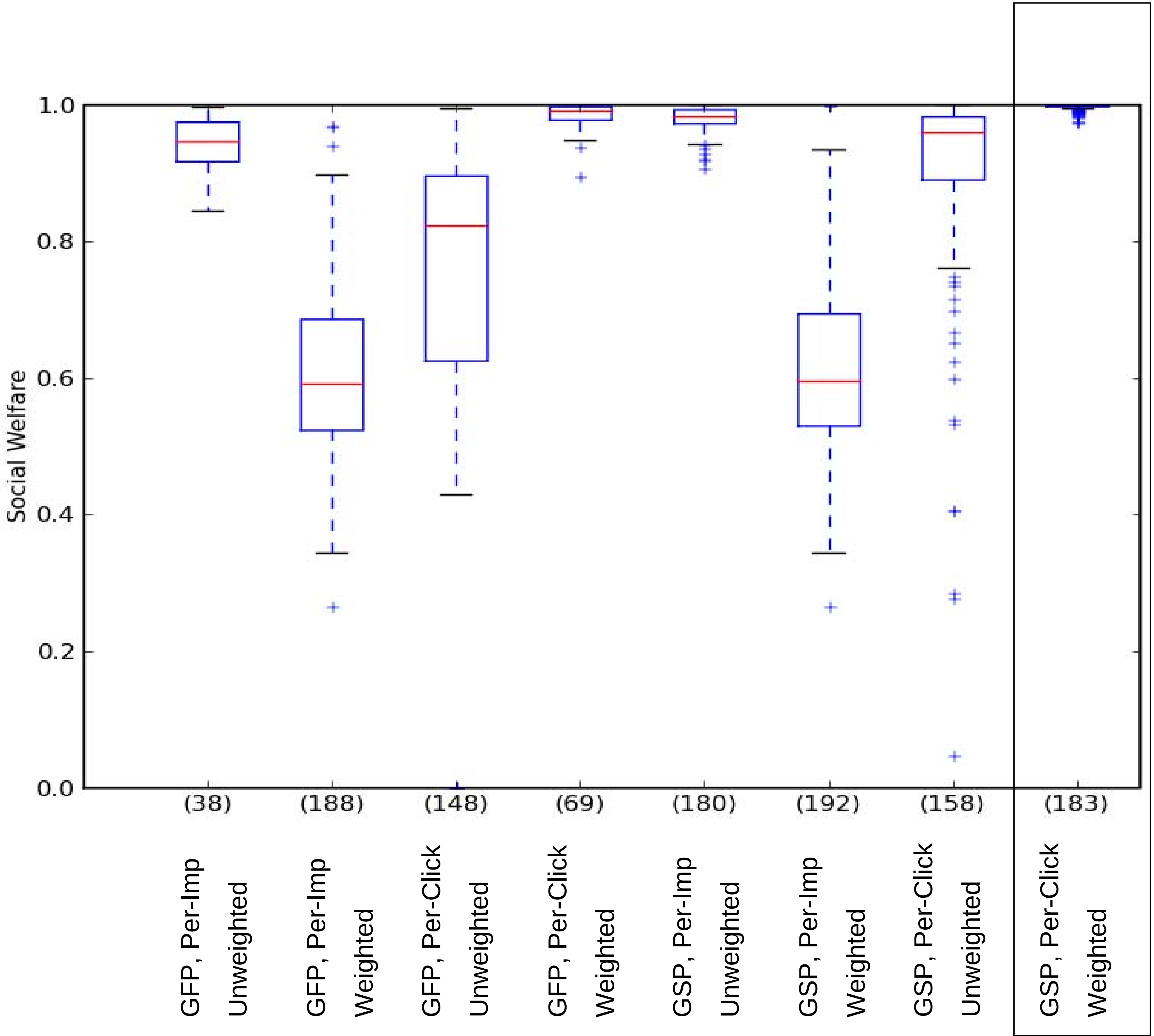
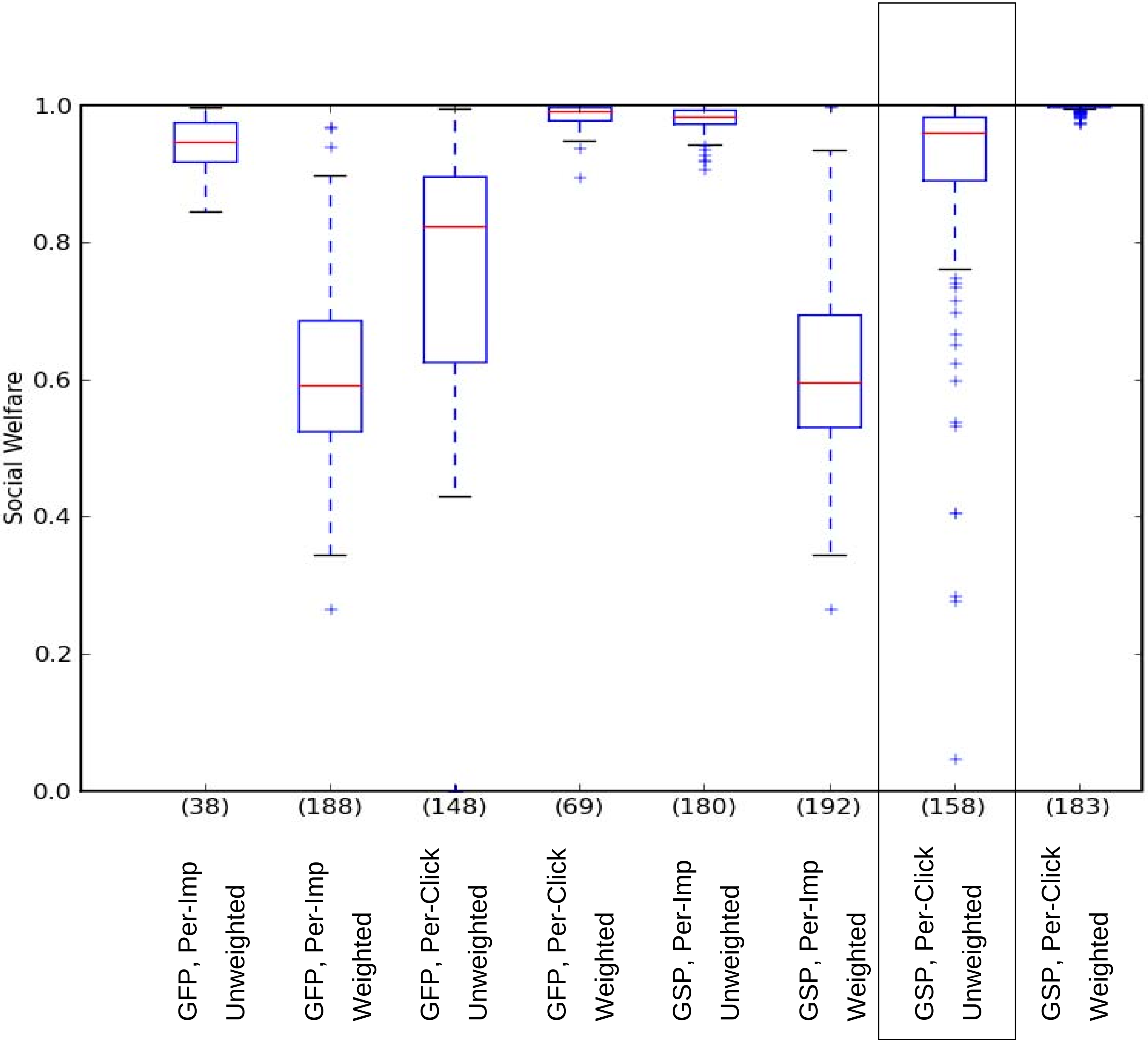


Social Welfare
1.0
0.8
0.6
0.4
0.2
0.0
(38)
(188)
(148)
(69)
(180)
(192)
(158)
(183)
GFP, Per-Imp Unweighted
GFP, Per-Imp Weighted
GFP, Per-Click Unweighted
GFP, Per-Click Weighted
GSP, Per-Imp Unweighted
GSP, Per-Imp Weighted
GSP, Per-Click Unweighted
GSP, Per-Click Weighted

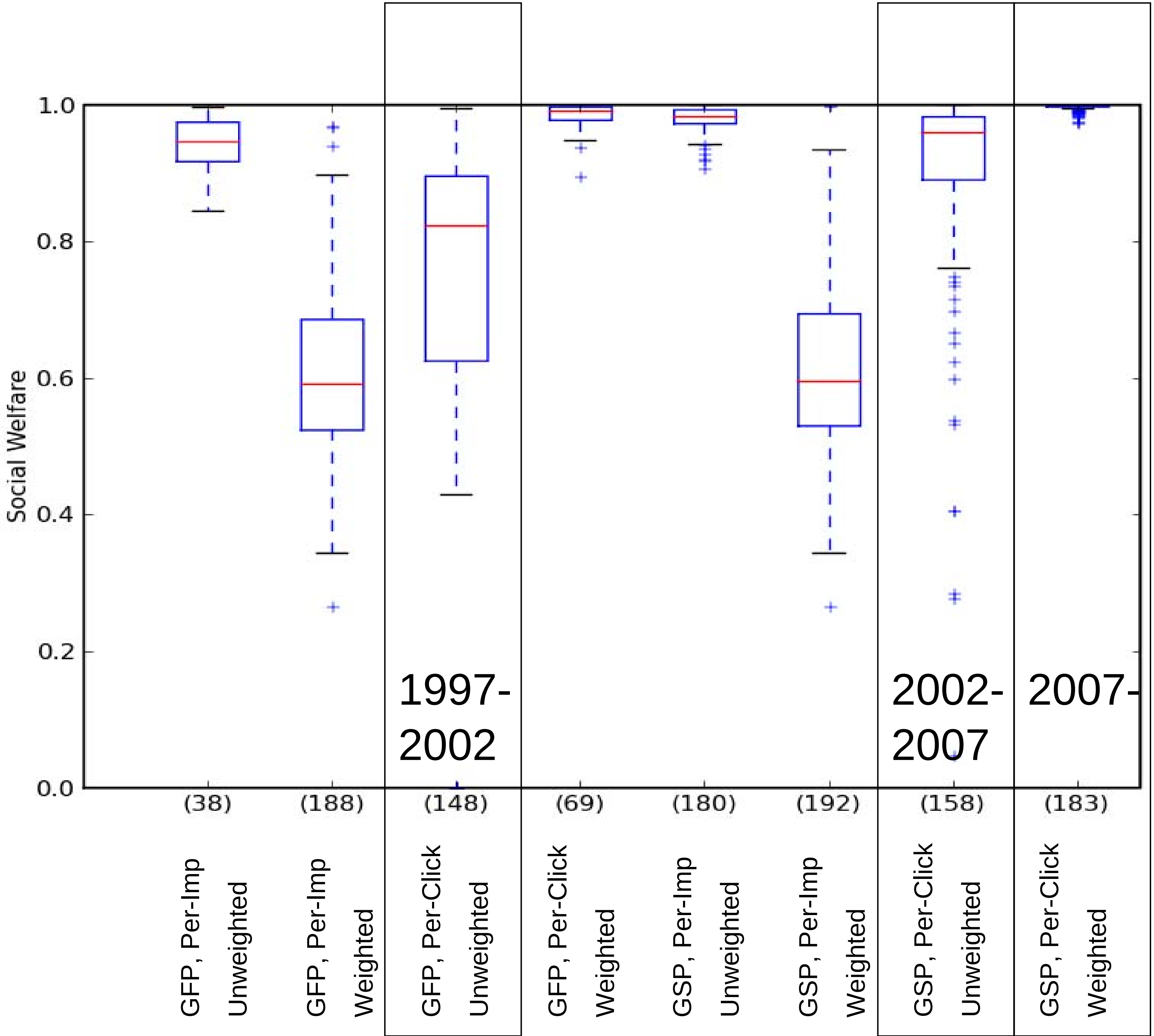
[Varian]: Any SNE gives rise to an efficient allocation



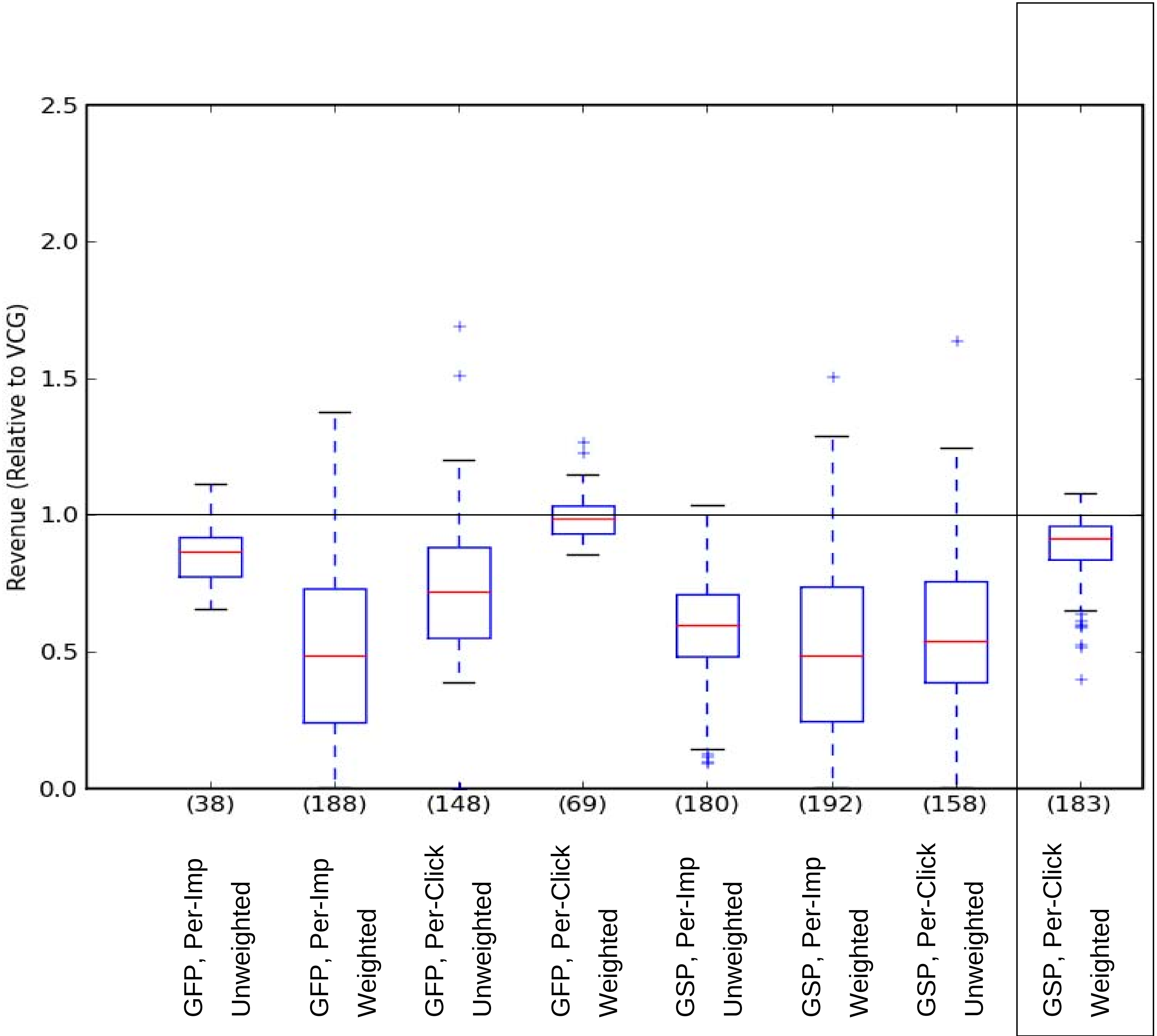
[EOS]'s auction with [Varian]'s preference model



Yahoo! Auctions: Past and Present

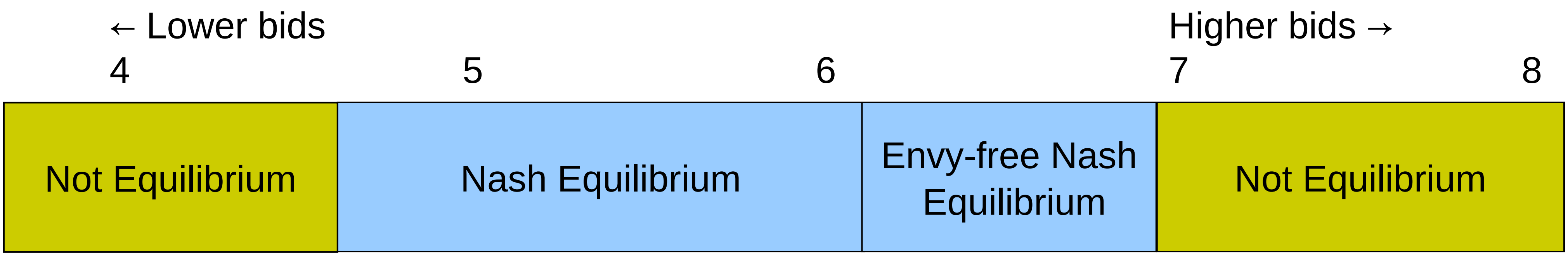


[Varian]: VCG revenue is a lower bound on SNE revenue



Multiple Equilibria of GSPs [Varian; EOS]

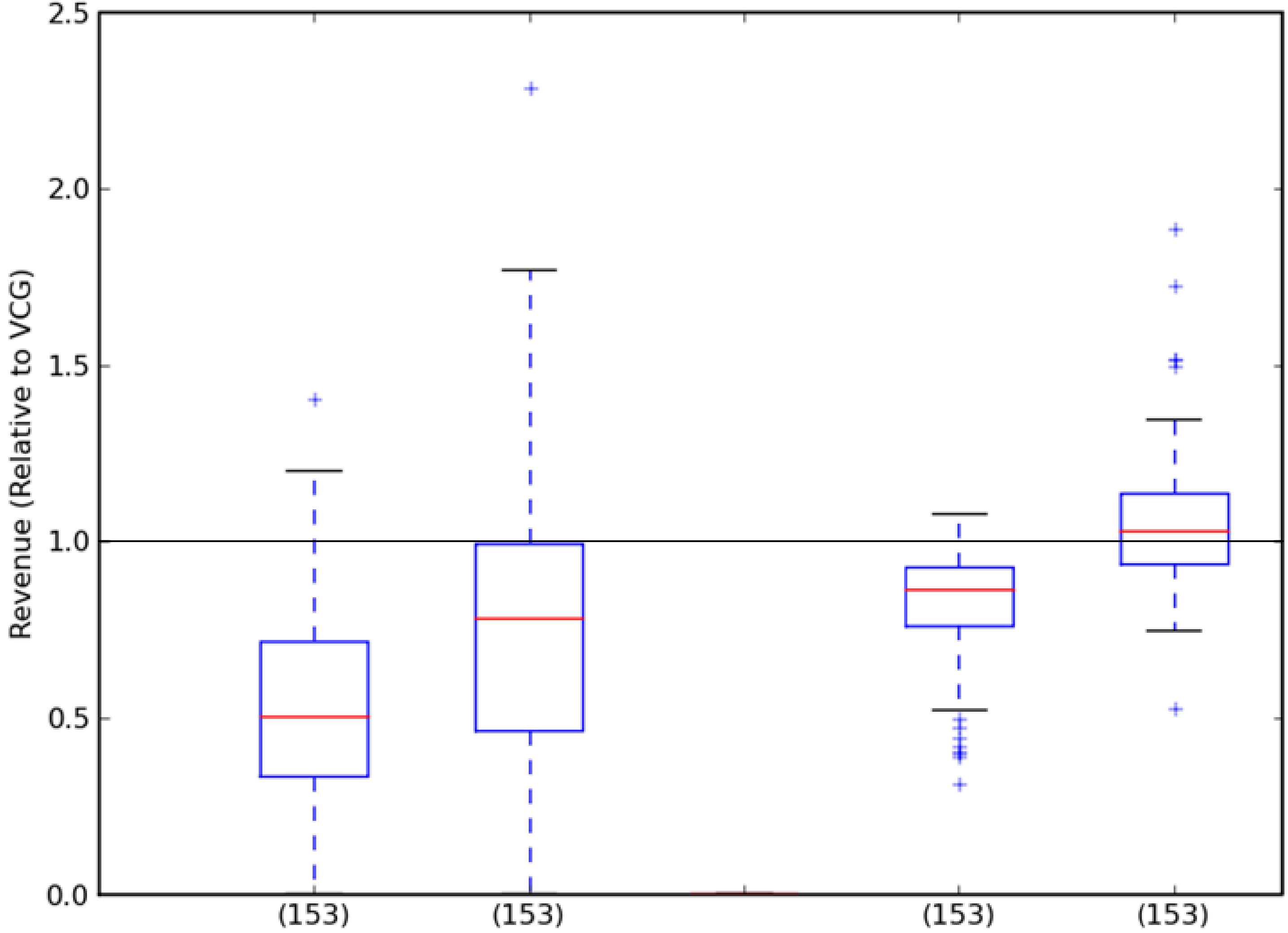
- Each agent can have many best responses to an equilibrium strategy profile.
  - Raising i's bid increases (i-1)'s price, decreasing i's envy.
- Given an envy-free NE / SNE, lowering an agent's bid may lead to an efficient, pure NE w/ sub-VCG revenue



- Even if pure NE exist for continuous bids, they may not exist for discrete bids.

Equilibrium selection

- Previous results simply showed the first equilibrium found by simpdiv
  - Often a mixed strategy over arbitrary points on equilibrium interval
- Local search approach to equilibrium selection:
  - Start point: Nash equilibrium found by simpdiv
  - Neighbours: *Nash equilibria* where one bid is changed by one increment
  - Objective: maximize/minimize sum of bids
  - Algorithm: Greedily raise bids (choose bidder by random permutation); random restarts.



Yahoo!

2002-2007

Google /

Yahoo! 2007-

Summary

- Many position auctions are tractable:
  - Polynomial-size AGG
  - Polynomial-time expected utility by dynamic programming
- Very general preference model:
  - Position-specific valuations
  - Non-separable CTRs (and arbitrary weights)
- Experimental results consistent with existing theory and practice.

Future Work

- Economic:
  - Use full preference model (learn from data)
  - Model richer preferences (e.g. cascading CTR [Aggarwal, et al, 2008; Kempe, Mahdian, 2008])
- Computational:
  - *In progress:* Adapt SEM [Porter, Nudelman, Shoham, 2006] to AGGs: Allows enumerating equilibria (answer questions like “what percentage of pure equilibria are envy free?”)

Thank You.